# Georgia Tech High School Math Day Logic/Puzzle Competition 

April 22, 2023

- Each correct answer is worth one point; there is no deduction for incorrect answers.
- Make sure to write your name and school (or N/A) clearly on the answer sheet
- You may use the test booklet as scratch paper, but no credit will be given for work in the booklet.
- You may keep the test booklet after the test has ended.

1. Color the two faces of a thin rectangle strip by red and blue, respectively. Twist one end of the strip by $180^{\circ}$, and then glue the two ends together: we have made a Möbius band. Cut the Möbius band along the middle circle that goes around the strip. How many colored segments are there in the resulting object, counting every face on every strip?

## Solution: 4

Cutting the Möbius band along the middle circle results in one two-sided strip, with each side having one red and one blue segments.
2. Place the numbers 1 to 5 once each into each row, column and region. Write the NW to SE diagonal on the answer sheet.

3. Place the numbers 1 to 5 once each into each row, column and region. Write the NW to SE diagonal on the answer sheet.

4. Place the numbers 1 to 5 once each into each row, column and region. Write the 4 th row on the answer sheet.

5. Place the numbers 1 to 5 once each into each row, column and region. Additionally, digits may not repeat on any diagonal. Write the 3rd row on the answer sheet.

| 2 | 1 | 5 | 4 | 3 |
| :--- | :--- | :--- | :--- | :--- |
| 5 | 4 | 3 | 2 | 1 |
| 3 | 2 | 1 | 5 | 4 |
| 1 | 5 | 4 | 3 | 2 |
| 4 | 3 | 2 | 1 | 5 |

6. With the same constraints as the previous problem, how many solutions are there to the following puzzle?


## Solution: 2

The sudoku/diagonal constraints mean that the NE corner cannot contain a 2,4 or 5. It was already established that there is a unique solution with a 3 in the NE corner. If there is a 1 instead, then there is a unique solution obtained by swapping all the 1 s and 3 s in the previous solution.
7. Place the numbers 1 to 6 once each into each row, column and region. Additionally, digits in the thermometers must increase from the bulb (circle) to the tip. Write the NW to SE diagonal on the answer sheet.

8. Consider the following 5 statements.
(A) There are exactly two true statements.
(B) Statement (C) and Statement (D) are both true or both false.
(C) Statement (D) and Statement (E) are both true or both false.
(D) Statement (A) and Statement (E) are both true or both false.
(E) Statement (C) is false.

Mark which statements are true on the answer sheet.

Solution: (A) and (C) are true.
If $(E)$ is true, then $(C)$ is false, then $(D)$ is false, then $(A)$ is false, then $(B)$ is false. But (B) cannot be false if we establish that both (C) and (D) are false.

Therefore (E) is false, so (C) is true, so (D) is false, so (A) is true, so (B) is false.
9. Place the numbers 1 to 5 once each into each row and column such that the numbers in each region either add or multiply together to the number listed, or subtract or divide together to the number listed.

Note: subtraction and division do not need to be left to right or up to down. A box reading 4 - can be filled with either 1,5 or 5,1 .

Write the NW to SE diagonal on the answer sheet.

| $\stackrel{15 \times}{5}$ | $3^{3-}$ | 1 | ${ }^{2-}$ | ${ }^{9+}$ |
| :---: | :---: | :---: | :---: | :---: |
| 3 | $10+$ 2 | 1- | 1 | 4 |
| ${ }^{1} 1$ | 5 | 4 | $11+$ 2 | 3 |
| $2 \div$ | 3 | ${ }^{2} 2$ | 5 | 4-1 |
| 2 | ${ }^{2-}$ | 3 | 4 | 5 |

10. Same rules as the previous puzzle but with the numbers 1 through 6 .

Write the NW to SE diagonal on the answer sheet.

11. Same rules as the previous puzzle.

Write the NW to SE diagonal on the answer sheet.

12. Aadhya, Chloe, El, Orion, Liu, and Magnus are running an obstacle course.

- Aadhya and Magnus finish 2 apart (not necessarily in that order)
- Aadhya, El and Magnus each finish before Chloe, who finishes before Orion
- El finishes before both Aadhya and Liu
- Liu and Orion do not finish next to each other
- neither do Aadhya and Chloe

Figure out the order in which the 6 finish. Write your answer as a sequence initials $A, C, E, O, L, M$ in the order in which they finish.

Solution: There were actually two solutions to this: EALMCO and MEALCO. The problem needed an extra hint to have a unique solution.
13. A 10 inch tall water bottle is filled up to a height of 6 inches when upright and to a height of 8 inches when upside down. What is the water:air ratio of the bottle? Assume that the bottom part of the bottle is cylindrical where the water sits.


Solution: $6: 2=3: 1$.
The water overlaps by 4 in . when putting the two orientations on top of each other. This means that the bottom 2 in . of water in the upright bottle is equal to the top 4 in . of air. So 2 in . of air compares to 6 in . of water. These are commensurable measurements because the bottom part of the bottle is a cylinder.
14. There are three boxes in front of you. One contains only white balls, one contains only black balls, and one contains an equal number of white and black balls. The boxes have each been labeled but incorrectly! For instance, the box labeled "white" might contain only black balls or might contain a mix but will not contain only white balls.

How many balls must be sampled in order to determine the correct labels?

## Solution: Just 1!

Sample the mixed box first. We know it isn't mixed so whatever color we draw must be the whole box. Lets say it's black. We know the mixed and black boxes weren't swapped because otherwise the white box would have white balls. Therefore, the black box contains white balls not mixed balls and that leaves the mixed balls in the white box.
15. Place the numbers 1 through 9 once each in a row such that number formed by

- digits 1 and 2 is divisible by 2
- digits 2 and 3 is divisible by 3
- digits 3 and 4 is divisible by 4
- and so on

Also, the first digit is smaller than the third digit.

|  | by 3 |  | by 5 |  | by 7 |  | by 9 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 8 | 7 | 2 | 5 | 4 | 9 | 6 | 3 |
| by 2 |  | by 4 |  | by 6 |  | by 8 |  |  |

16. Place the numbers 1 to 6 once each into each row, column and region. Additionally, groups of cells outlined with a dash (cages) must sum to the number in the top left of the cage. Numbers may not repeat within a cage. Write the SW to NE diagonal on the answer sheet.

17. How many unique paths are there from the SW corner to the NE corner using only East and North steps and avoiding the black squares?

| 1 | 1 | 3 | 3 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| 1 |  | 2 |  | 3 |
| 1 | 1 | 2 | 2 | 3 |
| 1 |  | 1 |  | 1 |
| 1 | 1 | 1 | 1 | 1 |

18. To find the correct 3 digit combination, you know

- 682 has one right digit and in the right place
- 614 has one right digit but in the wrong place
- 206 has two right digits but both in the wrong place
- 738 has all wrong digits
- 380 has one right digit but in the wrong place

What is the code?

Solution: 042
From the first two hints, we know that 6 is not a correct digit because its place does not change. We know that one of 8,2 and one of 1,4 are correct (but in the wrong places).

From the third hint, we know that 0 and 2 are correct digits (because 6 is incorrect).
From the fourth hint, we know that 3 and 8 are incorrect digits so that from the fifth hint we know that 0 is a correct digit, in the wrong place.

So 0 goes at the front since it is a correct digit and doesn't go in the other two places. We know 2 is correct and it goes at the end. The middle digit is either 1 and 4 but it has to be in the wrong place in the second clue so 4.

