Georgia Tech High School Math Day Group Competition April 22, 2023

Instructions: Write your names and schools (or N/A) below. Keep the booklet together. You may answer questions either by circling the correct answer (multiple choice) or by writing your answer on one of the pages on the back. Please indicate which question you are answering where you do so. Each group of questions in this section is based on a set of conditions. In answering some of the questions, it may be useful to draw a rough diagram.

(Questions 1–7) Dylan is hosting a party and invites nine fickle friends, some who get along, and some who don't. Five men—Ken, Lance, Matt, Nick, and Oscar—and four women—Pam, Reena, Sara, and Tracy—are invited. The following conditions apply:

- Tracy will come to the party only if Sara comes.
- If Oscar and Pam come, Lance will not come.
- Tracy is coming to the party unless both Lance and Reena come.
- Ken will come to the party if at least three women come.
- Nick will come to the party only if two or more women come.

Note: be careful not to assume things which aren't strictly written. For instance, "A only if B" means that if A happens then B for sure happens but does not mean that if B happens we can say anything about A necessarily.

Questions borrowed from https://www.cambridgelsat.com/resources/free-downloads/ logic-games-practice/ (Party Problem #34, Hard)

- 1. Which of the following could be a complete and accurate list of people who come to the party?
 - (A) Ken, Oscar, Pam, Tracy
 - (B) Lance, Ken, Oscar, Pam, Sara, Tracy
 - (C) Ken, Matt, Nick, Oscar, Pam, Sara, Tracy
 - (C) Matt, Oscar, Pam, Sara, Tracy
 - (D) Ken, Matt, Oscar, Pam, Sara
- 2. Each of the following could be true EXCEPT:
 - (A) Both Lance and Pam come to the party.
 - (B) Both Lance and Oscar come to the party.
 - (C) Both Lance and Sara come to the party.
 - (D) Neither Oscar nor Pam comes to the party.
 - (E) Neither Lance nor Sara comes to the party
- 3. What is the minimum number of people that will come to the party?
 - (A) 2
 - (A) 3
 - (B) 4
 - (C) 5

- (D) 6
- 4. If Nick comes to the party, which of the following must be false?
 - (A) Neither Lance nor Pam comes to the party.
 - (B) Neither Oscar nor Tracy comes to the party.
 - (C) Neither Pam nor Reena comes to the party.
 - (D) Neither Pam nor Sara comes to the party.
 - (D) Neither Oscar nor Pam comes to the party.
- 5. If both Oscar and Pam come to the party, each of the following must be true EXCEPT:
 - (A) Ken comes to the party.
 - (B) Lance does not come to the party.
 - (C) Tracy comes to the party.
 - (D) Sara comes to the party.
 - (E) Nick comes to the party.
- 6. If all four women come to the party, which of the following cannot be a complete list of men who come to the party?
 - (A) Ken, Matt, Nick, Oscar
 - (B) Ken, Lance, Matt
 - (C) Ken, Lance, Matt, Oscar
 - (C) Ken, Lance, Nick
 - (D) Ken, Oscar
- 7. Which of the following rules, if substituted for the condition that Nick will come to the party only if two or more women come, would have the same effect on determining who attends the party?
 - (A) If two or more women do not come to the party, Nick will not come.
 - (B) If Reena is the only woman at the party, Nick will not come.
 - (B) If Nick does not come to the party there are fewer than two women at the party.
 - (C) If two or more women come to the party, Nick will come.
 - (D) If Nick comes to the party, Tracy will come to the party.

Each group of questions in this section is based on a set of conditions. In answering some of the questions, it may be useful to draw a rough diagram.

(Questions 8–13) A toymaker will use exactly six different paint colors—fuchsia, goldenrod, indigo, jonquil, khaki, and mulberry—to paint three different model toys—an airplane, a race car, and a truck. For each toy, exactly two colors will be used and the colors will have one of two different roles—one color will be used as the base, and another will be used as a highlight. The following conditions apply:

- Fuchsia will be used as a base.
- Goldenrod and mulberry cannot be used on the same toy.
- Indigo will be used on the race car.
- If mulberry is used on the airplane, goldenrod will be used on the race car.
- If jonquil is used as a base, mulberry will be used as a highlight.

Questions borrowed from https://www.cambridgelsat.com/resources/free-downloads/ logic-games-practice/ (The Toy Maker Game #35, Hard)

- 8. Which of the following could accurately reflect the colors used for each toy?
 - (A) Airplane: base: mulberry, highlight: jonquil Race car: base: fuchsia, highlight: goldenrod Truck: base: khaki, highlight: indigo
 - (B) Airplane: base: mulberry, highlight: jonquil Race car: base: fuchsia, highlight: indigo Truck: base: goldenrod, highlight: khaki
 - (C) Airplane: base: goldenrod, highlight: jonquil Race car: base: fuchsia, highlight: indigo Truck: base: mulberry, highlight: khaki
 - (C) Airplane: base: jonquil, highlight: goldenrod Race car: base: fuchsia, highlight: indigo Truck: base: mulberry, highlight: khaki
 - (D) Airplane: base: goldenrod, highlight: mulberry Race car: base: fuchsia, highlight: indigo Truck: base: jonquil, highlight: khaki
- 9. Which of the following CANNOT be the complete list of colors used as highlights?
 - (A) goldenrod, indigo, mulberry
 - (B) jonquil, indigo, khaki
 - (C) goldenrod, jonquil, khaki

- (D) goldenrod, indigo, khaki
- (D) goldenrod, indigo, jonquil

10. If goldenrod is used for the truck, for which toys will the pair of colors be determined?

- (A) only the airplane
- (B) only the race car
- (B) only the truck
- (C) only the airplane and the truck
- (D) only the race car and the truck
- 11. If mulberry is used as the base for the truck, which of the following represents the complete list of colors that could be used as a highlight for the truck?
 - (A) khaki
 - (B) jonquil, khaki
 - (B) goldenrod, jonquil, khaki
 - (C) fuchsia, jonquil, khaki
 - (D) goldenrod, indigo, khaki
- 12. If jonquil and mulberry are used to paint the airplane, each of the following could be true EXCEPT:
 - 1. Goldenrod is used as the base of the race car.
 - 2. Goldenrod is used as the highlight of the race car.
 - 3. Indigo is used as the base of the race car.
 - 4. Fuchsia is used as the base of the truck.
 - (E) Khaki is used as the base of the truck.
- 13. Which of the following, if substituted for the rule that if jonquil is used as a base, mulberry will be used as a highlight, would have the same effect in determining where colors can and cannot be used?
 - (A) If mulberry has a different role from fuchsia, jonquil will be used as a base.
 - (B) If jonquil has a different role from fuchsia, mulberry will be used as a base.
 - (C) Jonquil and mulberry will be used for different roles.
 - (D) Mulberry will not be used as a base unless jonquil is used as highlight.
 - (D) Unless mulberry is used as a base, jonquil will be used as a base.

- 14. 5 pirates of different levels of seniority are to split a treasure of 100 gold coins. They decide to split the coins according to the following scheme:
 - The oldest pirate proposes how to share the coins, and ALL pirates (including the oldest) vote for or against it.
 - If 50% or more of the pirates vote for it, then the coins will be shared that way. Otherwise, the pirate proposing the scheme will be thrown overboard, and the process is repeated with the pirates that remain.
 - As pirates tend to be a bloodthirsty bunch, if a pirate would get the same number of coins if he voted for or against a proposal, he will vote against so that the pirate who proposed the plan will be thrown overboard.
 - Assuming that all 5 pirates are intelligent, rational, greedy, and do not wish to die, (and are rather good at math for pirates) what will happen?

Problem borrowed from https://www.mathsisfun.com/puzzles/5-pirates.html.

Our Solution:

The oldest pirate will propose a 98 : 0 : 1 : 0 : 1 split, in other words the oldest pirate gets 98 coins, the middle pirate gets 1 coin and the youngest gets 1 coin.

Let us name the pirates (from oldest to youngest): Alex, Billy, Colin, Duncan and Eddie.

Working backwards:

2 Pirates: Duncan splits the coins 100 : 0 (giving himself all the gold). His vote (50%) is enough to ensure the deal.

3 Pirates: Colin splits the coins 99 : 0 : 1. Eddie will accept this deal (getting just 1 coin), because he knows that if he rejects the deal there will be only two pirates left, and he gets nothing.

4 Pirates: Billy splits the coins 99 : 0 : 1 : 0. By the same reasoning as before, Duncan will support this deal. Billy would not waste a spare coin on Colin, because Colin knows that if he rejects the proposal, he will pocket 99 coins once Billy is thrown overboard. Billy would also not give a coin to Eddie, because Eddie knows that if he rejects the proposal, he will receive a coin from Colin in the next round anyway.

5 Pirates: Alex splits the coins 98 : 0 : 1 : 0 : 1. By offering a gold coin to Colin (who would otherwise get nothing) he is assured of a deal.

(Note: In the final deal Alex would not give a coin to Billy, who knows he can pocket 99 coins if he votes against Alex's proposal and Alex goes overboard. Likewise, Alex would not give a coin to Duncan, because Duncan knows that if he votes against the proposal, Alex will be voted overboard and Billy will propose to offer Duncan the same single coin as Alex. All else equal, Duncan would rather see Alex go overboard and collect his one coin from Billy.)

15. What happens in the previous scenario if the oldest pirate is not included in the vote and only the other pirates will vote?

Problem borrowed from https://www.mathsisfun.com/puzzles/5-pirates-version-2. html.

Our Solution:

The oldest pirate will propose a 97 : 0 : 1 : 0 : 2 split.

Let us name the pirates (from oldest to youngest): Alex, Billy, Colin, Duncan and Eddie.

Working backwards:

2 Pirates: Duncan splits the coins 0: 100 (giving all to Eddie). Otherwise, and perhaps even then, Eddie would vote against him and over he goes!

3 Pirates: Colin splits the coins 99 : 1 : 0. Eddie is going to vote against him no matter what (see above) so gets nothing, but Duncan will vote for him, to get at least one gold out of it (if Duncan votes against him, there will only be two pirates remaining and Duncan will get nothing, and may even lose his life!)

4 Pirates: Billy splits the coins 97 : 0 : 2 : 1. This way, Eddie will vote for him, and so will Duncan - they're getting more than they would under 3 pirates.

5 Pirates: Alex splits the coins 97 : 0 : 1 : 0 : 2. This way, Eddie will vote for him, and so will Colin - they're both getting better than they would under 4 pirates.

- 16. Albert and Bernard just became friends with Cheryl, and they want to know when her birthday is. Cheryl gives them a list of 10 possible dates
 - May 15, May 16, May 19
 - June 17, June 18
 - July 14, July 16
 - August 14, August 15, August 17

Then in secret Cheryl tells Albert the correct month (without the day) and Bernard the correct day (without the month).

Following this, a conversation unfolds

Albert- I don't know when Cheryl's birthday is, but I know that Bernard doesn't know too.

Bernard- At first I didn't know when Cheryl's birthday was, but I now know.

Albert- Then I also know when Cheryl's birthday is.

When is Cheryl's birthday?

Solution: Note- This problem was taken from the 2015 SASMO competition and has a wikipedia page called Cheryl's Birthday.

The only possibility for Bernard to know the exact birthday would be if he was told the day was 18 or 19, since these only appear once.

For Albert to know that Bernand does not know, Albert must have been told either July or August (as these rule out 18 and 19).

Then after Bernard concludes that August and July are the only viable options, if he is able to make a complete determination of the date he must have been told 15,16, or 17 (as 14 would leave ambiguity).

Then knowing that July 16, August 15, and August 17 are the only possible dates, in order for there to be a final determination he must have been told July. Therefore Cheryl must have the birthday of July 16.

17. Place the numbers 1 to 9 once each into each row and column such that the numbers in each region either add or multiply together to the number listed, or subtract or divide together to the number listed.

Note: subtraction and division do not need to be left to right or up to down. For example, a box reading 4- can be filled with either 1, 5 or 5, 1.

¹⁷⁺ 3	9	192× 4	6	^{2÷} 2	16+ 1	280× 5	7	8
5	¹ 3	^{30×} 6	8	4	7	^{14×} 2	°- 9	1
4÷ 2	4	5	^{108×}	3	8	7	1	¹⁵⁺ 6
8	²⁶⁺	9	4	¹⁵⁺ 5	²– 3	1	2	7
⁸⁺ 1	8	3	⁷ 7	9	^{24×} 6	4	₃ 5	2
7	13+ 2	8	3	1	^{10×} 5	^{54×}	^{2÷} 6	₅_ 4
28× 4	7	¹ 1	₃ 5	8	2	6	3	9
²⁰⁺ 9	1	98× 7	48× 2	6	4	¹⁵⁺ 3	8	² — 5
6	5	2	1	7	^{1—} 9	8	4	3